Synthese (2008) 162:31–35
DOI 10.1007/s11229-007-9170-0

Could there be exactly two things?

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Received: 22 March 2006 / Accepted: 17 April 2007 / Published online: 8 June 2007

Abstract Many philosophers think that, necessarily, any material objects have a fusion (let's call that doctrine "Universalism"). In this paper I point out a couple of strange consequences of Universalism and related doctrines, and suggest that they are strange enough to constitute a powerful argument against those views.

Keywords Ontology · Universalism · Mereology

By "thing" I mean *material* thing. We all believe that there actually are many more than two things, but could there have been exactly two? More generally, what are the *n*s such that there could have been exactly *n* things?

Pre-theoretically, the following conjecture is quite attractive:

Primitive Cardinality: For any *n*, there could have been exactly *n* things.

For instance, we naturally judge that there could have been exactly two things, and exactly three, . . . Attractive as it is, many philosophers are committed to denying Primitive Cardinality. For instance, many philosophers believe in

Universalism: Necessarily, any material objects have a fusion.[1]

[1] The notion of fusion is defined in terms of the notion of part in one usual way: *x* is a *fusion* of the Fs if and only if (*a*) each of the Fs is part of *x*, and (*b*) every part of *x* overlaps at least one of the Fs (*x* and *y overlap* if and only if they have a part in common). (For a different definition of fusion, see the end of note 4.) The view is called "Universalism" following the terminology of van Inwagen (1990), p. 74 (although van Inwagen is concerned with composition instead of fusion, where the difference is that some things compose another only if they don't overlap).

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It can be shown that Universalism implies that there could not have been exactly two things, given the following minimal assumptions:

Reflexivity: Necessarily, everything is a part of itself.
Principle of Weak Supplementation: Necessarily, if x is a proper part of y then some part of y does not overlap with x (x is a *proper part* of y just in case x is part of y and $x \neq y$).[2]

Notice that these three principles (Universalism, Reflexivity and the Principle of Weak Supplementation) are substantially weaker that Classical Extensional Mereology (CEM). Many philosophers believe not only that any objects have a sum, but also that any objects have a *unique* sum. In other words, many philosophers believe not only in Universalism but also in:

Uniqueness: Necessarily, if x and y are sums of the Fs, then $x = y$.[3]

Indeed, given only the following principle

Transitivity: Necessarily, if x is part of y and y is part of z, then x is part of z;

[2] *Proof*: Suppose for *reductio* that the world consists of a and b and that $a \neq b$. By Universalism there is a fusion c of a and b. By our initial assumption $c = a$ or $c = b$. Suppose with no loss of generality that $c = a$. Since $c = a$, $a \neq b$ and b is a part of c, b is a proper part of a. It follows from the Principle of Weak Supplementation that there is an object d such that d is part of a and does not overlap b. By reflexivity, every object overlaps with itself, but b doesn't overlap with d, so $b \neq d$. Since a has b as a part and d does not, $a \neq d$. This means that a, b and d are all different, contradicting our initial assumption. Notice also that the proof will still go through even if we replaced the Principle of Weak Supplementation with the still weaker Principle of Weak Company (see Simons (1987), pp. 26–28):

Principle of Weak Company: Necessarily, if x is a proper part of y then there is a proper part of y that is distinct from x.

The proof would *not* go through under still weaker principles. For instance, consider the following (the principle, although not the name, is due to an anonymous referee for *Synthese*):

Principle of Too Weak Complementation: Necessarily, if x is a proper part of y but y is not part of x, then there is a part of y that doesn't overlap x.

Here is a model with exactly two objects that satisfies Reflexivity and the Principle of Too Weak Complementation: there are two objects, a and b, where a is a proper part of b and b is proper part of a, and each is a part of itself. But it should be noticed that if we assume that the parthood relation is antisymmetric (i.e, that if x is part of y and y is part of x, then $x = y$), then the Principle of Too Weak Complementation also entails that there couldn't have been exactly two things. It is worth noting that some authors (such as Simons (1987)) believe that the Principle of Weak Supplementation is part of the meaning of "part." See also note 3.

[3] It is common to point out that philosophers who believe that there can be two different objects that share all of their parts are committed to denying Uniqueness. It is interesting to note that they are also committed to denying Weak Supplementation. For suppose that a statue and the lump of clay that compose it share all of their parts but are different. By Reflexivity, the (whole) lump is a part of the lump, and so it is part of the statue. But given that the statue and the lump are different, the lump is a proper part of the statue. By Weak Supplementation, there must be a part of the statue that doesn't overlap the lump, contradicting the assumption that the lump and the statue share all of their parts. The Principle of Weak Company (see note 2) is compatible with both Reflexivity and the existence of two different things that share all of their parts, as long as those things have proper parts (for instance, consider a model with exactly three things, all of which are part of each other). The Principle of Too Weak Complementation is compatible with both Reflexivity and the existence of two different things that share all of their parts, even when those two things are all that exists (see the model in note 2).

Weak Supplementation and Universalism entail Uniqueness.[4] Moreover, Transitivity, Universalism and Uniqueness are one common axiomatic basis for CEM, whose only finite models are of size $2^n - 1$, where n is the number of atoms (objects without proper parts) in the model.[5] In the presence of Transitivity, then, Universalism and Weak Supplementation entail not only that there couldn't have been exactly two objects, but also that there couldn't have been exactly five, exactly 150, etc. That is, CEM entails massive violations of Primitive Cardinality.

CEM has been criticized on the basis that it is too restrictive (because it doesn't allow for the existence of objects that share all of their parts) and also on the basis that it is too liberal (because it entails that there is a sum of, e.g., my nose and the Eiffel Tower). The present criticism has something in common with the first one, because it is also a criticism to the effect that CEM is too restrictive. The difference is that the criticism is not that there is a particular *kind* of object that is prohibited, but that there are particular *numbers* of objects that are prohibited.[6]

We have an apparent conflict, then, between the pre-theoretical appeal of Primitive Cardinality on the one hand and CEM on the other. How can this apparent conflict be resolved? There are three possible resolutions. The first one is to claim that the conflict is *merely* apparent, and that both Primitive Cardinality and Classical Extensional Mereology are correct. If, for example, there are two notions of 'thing', say $thing*$ and $thing\dagger$, there is no conflict between Primitive Cardinality and CEM if one of them is interpreted as referring to $thing*$s and the other as referring to $thing\dagger$s.[7] The other two resolutions take the appearance of conflict at face value. One of them consists in claiming that Primitive Cardinality is false, and the other in claiming that CEM is false instead.

I have little to say about the first resolution. The general form of the strategy has a distinguished philosophical pedigree, but it is suggestive that many participants in ontological

[4] *Proof*: Suppose for *reductio* that there are some Fs that have two different sums, a and b. Then, by Universalism, there is a sum c of a and b. Now, either a and b are related by the parthood relation to each other or they are not. Case 1: a is not part of b and b is not part of a. Thus, by part (a) of the definition of fusion both a and b are proper parts of c. Therefore, by Weak Supplementation there is a part of c, d, that doesn't overlap, say, a. But, by part (b) of the definition of fusion, every part of c overlaps either a or b. Since d doesn't overlap a it must overlap b. By the definition of overlap, this means that there is an object e which is part of d and part of b. But, again by part (b) of the definition of fusion, every part of b overlaps some F. So e overlaps some F. This means that there is an object f which is part of e and part of some F. But every F is part of a. So, by Transitivity, f is part of a. But e is part of d. So, by Transitivity, f is part of d. It follows that a overlaps d, contradicting our earlier conclusion that it doesn't. Case 2: either a is part of b or b is part of a. Without loss of generality, suppose that a is part of b. Given that a and b are different, a is a proper part of b. Therefore, by Weak Supplementation there is a part c of b that doesn't overlap a. But, by part (b) of the definition of fusion, every part of b overlaps some F. So there is an object d which is part of c and part of some F. But, by part (a) of the definition of fusion, every F is part of a. So, by Transitivity, d is part of a. Since d is part of c, this means that c overlaps a, contradicting our earlier conclusion that it doesn't.

Interestingly, this proof doesn't work if instead of the definition of fusion given in note 1 we use another popular definition, namely, that x is a $fusion$ of the Fs if and only if, for every y, y overlaps x if and only if it overlaps some F. The definition offered in note 1 is used, for example, by Lewis (1991), whereas the one just given is used, for example, by Simons (1987).

[5] Cf. Simons (1987), p. 17.

[6] It turns out that CEM conflicts not only with Primitive Cardinality, but also with set theory: Uzquiano (2006) has shown that, under plausible assumptions, set theory's only infinite models have strongly innaccessible cardinality, whereas CEM doesn't have infinite models of that cardinality. Moreover, this conflict doesn't go away if one thinks that the mereological principles are *contingently* true.

[7] The believer in CEM can then understand the assertions of the believer in Primitive Cardinality as being made with respect to a restricted domain, and the believer in Primitive Cardinality can understand the assertions of the believer in CEM as involving plural quantification over objects in the domain.

disputes resist this kind of ecumenical re-interpretation of their position. If so, charity bids us to consider the other options.

If the choice, then, is between Primitive Cardinality and CEM, who wins? One consideration in favor of Primitive Cardinality runs along the following lines. The support for Primitive Cardinality derives from our particular pre-theoretical judgments that there could have been exactly two things, and exactly three things, and …The support for CEM comes instead from the simplicity and appeal of its general principles. Standard methodological procedures in other areas of philosophy give precedence to particular pre-theoretic judgments over general theoretical principles,[8] and that is *prima facie* evidence in favor of Primitive Cardinality.

The argument shouldn't be over-generalized. After all, we abandoned our pre-theoretic judgments about simultaneity when they were seen to be in conflict with a well-confirmed physical theory. Couldn't the defender of CEM claim that we should take the same attitude here? There is an important difference between the Special Theory of Relativity and CEM that undermines this reply, however. Whereas the Special Theory of Relativity is, as just said, a well-confirmed empirical theory, it is not even clear what purpose CEM is supposed to serve. Its history is closely related to dissatisfaction of some kind or another with set theory. Leśniewski created it with the aim of identifying what he thought was a mistake in the reasoning leading to Russell's paradox.[9] For Goodman, it was a nominalistically respectable *replacement* for set theory.[10] For Lewis, it provided (together with the notion of singleton) the foundations for contemporary set theory.[11] For those philosophers free of nominalistic scruples, however, it is not clear what work there is for CEM to do.[12] The defender of CEM could say that there is an obvious work for it to do: to provide a theory of the part-whole relation. But what I have been arguing is precisely that CEM cannot be a *true* theory of the part-whole relation.

Acknowledgements Thanks above all to Agustín Rayo for invaluable help. Many thanks too to Manuel Comesaña, Antony Eagle, Ned Markosian, Carolina Sartorio, Alan Sidelle, Gabriel Uzquiano and two anonymous referees for *Synthese*.

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[8] Thus, for instance, most contemporary epistemologists are in practice (if not always in theory) *particularists* in the sense of Chisholm (1973).

[9] For a brief historical account of Leśniewski's motivations, see Simons (1987), pp. 101–104.

[10] See Goodman (1956).

[11] See Lewis (1991).

[12] Van Cleve (1986) and Lewis (1986) have argued that if we accept the existence of *some* fusions then we have to accept Universalism, on pain of arbitrariness (Van Cleve) or committing ourselves to ontological vagueness (Lewis). If they are right, then there is a job for Universalism to do: to save us from arbitrariness or ontological vagueness. It remains true, however, that the conflict is now between a principle with a great deal of pre-theoretical appeal (Primitive Cardinality) and complicated theoretical arguments to the effect that the principle is wrong. Moreover, Van Cleve's and Lewis' arguments have been forcefully resisted—see, for instance, Markosian (1998) and Merricks (2005).

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